5^{th} Iranian Geometry Olympiad

Elementary Level

Thursday, September 6, 2018

The contest problems are to be kept confidential until they are posted on the official IGO website: http://igo-official.ir .

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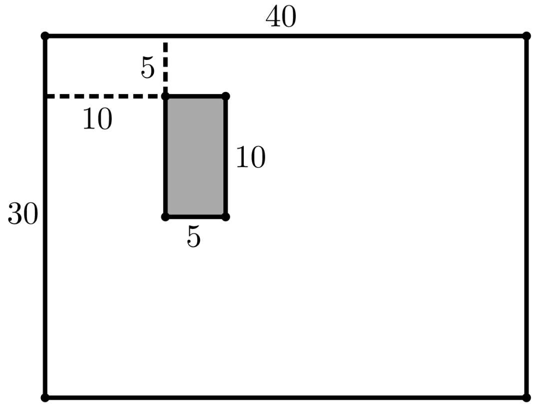
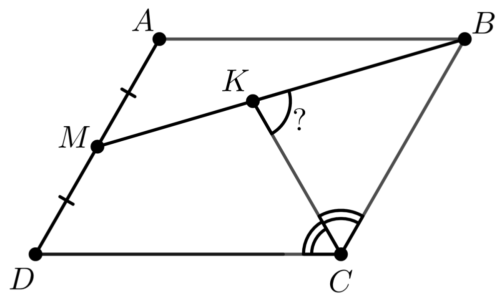
1 As shown below, there is a 40×30 paper with a filled 10×5 rectangle inside of it. We want to cut out the filled rectangle from the paper using four straight cuts. Each straight cut is a straight line that divides the paper into two pieces, and we keep the piece containing the filled rectangle. The goal is to minimize the total length of the straight cuts. How to achieve this goal, and what is that minimized length? Show the correct cuts and write the final answer. There is no need to prove the answer.

Seperti terlihat di bawah, terdapat sebuah kertas berukuran 40×30 yang di dalamnya memuat persegi panjang yang diarsir berukuran 10×5. Kita ingin mendapatkan persegi panjang yang diarsir dari kertas tersebut dengan empat kali pemotongan. Masing-masing pemotongan adalah sebuah garis lurus yang membagi kertas menjadi dua bagian, dan kita menyimpan bagian kertas yang mengandung persegi panjang yang diarsir. Tujuan yang diinginkan adalah untuk membuat total panjang garis pemotongan seminimal mungkin. Bagaimana untuk mencapai tujuan tersebut dan berapa panjang minimalnya? Tunjukkan cara pemotongan yang benar dan tuliskan panjang minimal yang diinginkan. Anda tidak perlu membuktikan jawaban anda.



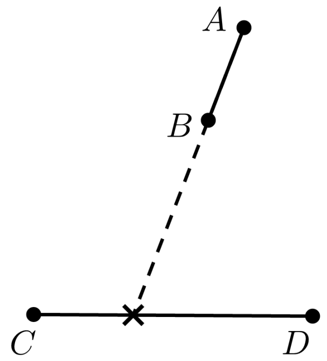
2 Convex hexagon $A_1A_2A_3A_4A_5A_6$ lies in the interior of convex hexagon $B_1B_2B_3B_4B_5B_6$ such that $A_1A_2 \parallel B_1B_2$, $A_2A_3 \parallel B_2B_3$,..., $A_6A_1 \parallel B_6B_1$. Prove that the areas of simple hexagons $A_1B_2A_3B_4A_5B_6$ and $B_1A_2B_3A_4B_5A_6$ are equal. (A simple hexagon is a hexagon which does not intersect itself.)

3 In the given figure, $ABCD$ is a parallelogram. We know that $\angle D = 60°$, $AD = 2$ and $AB = \sqrt{3}+1$. Point M is the midpoint of AD. Segment CK is the angle bisector of C. Find the angle K.

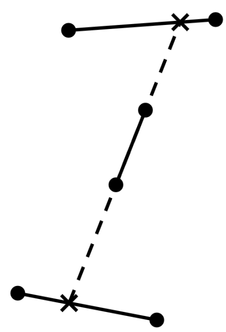


4 Circle ω is given on the plane. Two circles with centers O_1, O_2 lie inside of ω and are tangent to it. Chord AB of ω is tangent to these two circles such that these two circles lie on opposite sides of this chord. Prove that $\angle O_1AO_2 + \angle O_1BO_2 > 90°$.

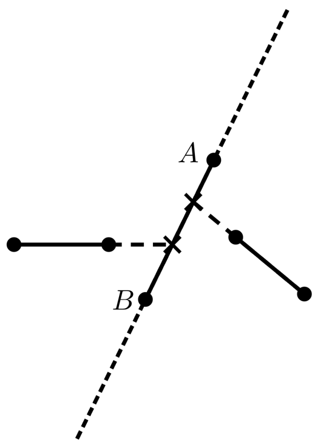
5 There are some segments on the plane such that no two of them intersect each other (even at the ending points). We say segment AB **breaks** segment CD if the extension of AB cuts CD at some point between C and D.



(a) Is it possible that each segment when extended from both ends, breaks exactly one other segment from each way?



(b) A segment is called **surrounded** if from both sides of it, there is exactly one segment that breaks it. (*e.g.* segment AB in the figure.) Is it possible to have all segments to be surrounded?



Time: 240 minutes.
Each problem is worth 8 points.