5^{th} **Iranian Geometry Olympiad**

Advanced Level

Thursday, September 6, 2018

The contest problems are to be kept confidential until they are posted on the official IGO website: http://igo-official.ir .

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1 Two circles ω_1, ω_2 intersect each other at points A, B. Let PQ be a common tangent line of these two circles with $P \in \omega_1$ and $Q \in \omega_2$. An arbitrary point X lies on ω_1. Line AX intersects ω_2 for the second time at Y. Point $Y' \neq Y$ lies on ω_2 such that $QY = QY'$. Line $Y'B$ intersects ω_1 for the second time at X'. Prove that $PX = PX'$

2 In acute triangle ABC, $\angle A = 45°$. Points O, H are the circumcenter and the orthocenter of ABC, respectively. D is the foot of altitude from B. Point X is the midpoint of arc AH of the circumcircle of triangle ADH that contains D. Prove that $DX = DO$.

3 Find all possible values of integer $n > 3$ such that there is a convex n-gon in which, each diameter is the perpendicular bisector of at least one other diameter.

4 Quadrilateral $ABCD$ is circumscribed around a circle. The angle bisectors of angles between diameters AC, BD intersect the segments AB, BC, CD, DA at points K, L, M and N. Given that $KLMN$ is cyclic, prove that so is $ABCD$.

5 $ABCD$ is a cyclic quadrilateral. A circle passing through A, B is tangent to segment CD at point E. Another circle passing through C, D is tangent to AB at point F. Point G is the intersection point of AE, DF, and point H is the intersection point of BE, CF. Prove that the incenters of triangles AGF, BHF, CHE, DGE lie on a circle.

Time: 270 minutes.

Each problem is worth 8 points.