5^{th} **Iranian Geometry Olympiad**

Intermediate Level

Thursday, September 6, 2018

The contest problems are to be kept confidential until they are posted on the official IGO website: http://igo-official.ir .

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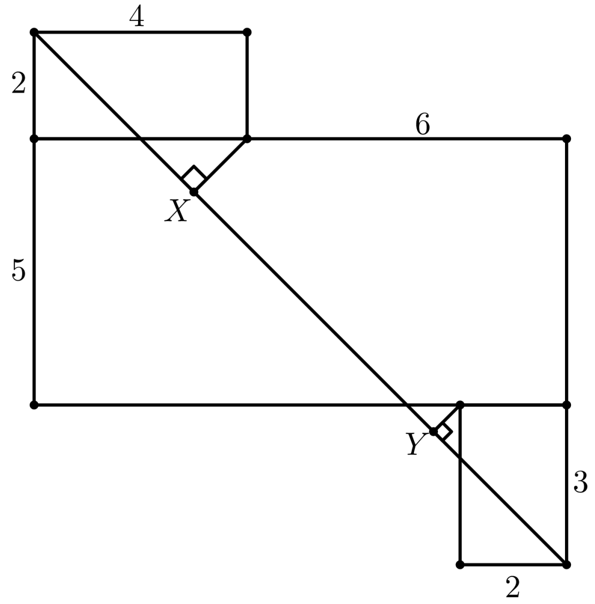
1 There are three rectangles in the following figure. The lengths of some segments are shown. Find the length of the segment XY.



2 In convex quadrilateral $ABCD$, the diameters AC and BD meet at the point P. We know that $\angle DAC = 90°$ and $2\angle ADB = \angle ACB$. If we have $\angle DBC + 2\angle ADC = 180°$ prove that $2AP = BP$.

3 Let ω_1, ω_2 be two circles with centers O_1 and O_2, respectively. These two circles intersect each other at points A and B. Line O_1B intersects ω_2 for the second time at point C, and line O_2A intersects ω_1 for the second time at point D . Let X be the second intersection of AC, ω_1. Also Y is the second intersection point of BD, ω_2. Prove that $CX = DY$.

4 We have a polyhedron all faces of which are triangle. Let P be an arbitrary point on one of the edges of this polyhedron such that P is not the midpoint or endpoint of this edge. Assume that $P_0 = P$. In each step, connect P_i to the centroid of one of the faces containing it. This line meets this face again at point P_{i+1}. Continue this process with P_{i+1} and the other face containing P_{i+1}. Prove that by continuing this process, we cannot pass through all the faces. (The centroid of a triangle is the point of intersection of its medians.)

5 Suppose that $ABCD$ is a parallelogram such that $\angle DAC = 90°$. Let H be the foot of perpendicular from A to DC, also let P be a point along the line AC such that the line PD is tangent to the circumcircle of the triangle ABD. Prove that $\angle PBA = \angle DBH$.

Time: 270 minutes.

Each problem is worth 8 points.